Notes on alternatives to MOST

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1 MOST is wrong but widely used

There is no doubt that MOST is wrong. It fails in the limit of free convection. It predicts wrong forms of the wind and temperature profiles in unstable surface layers and fails to collapse spectra onto universal curves in unstable surface layers. Yet MOST is widely used. Why?

The first answer is that there has been no accepted alternative. Atmospheric stability has large effects on boundary layer processes so it must be modelled somehow. Secondly, MOST has been widely 'calibrated' using observational data, and these results are consistent enough that there is consensus on the 'best' constants to use in the various MOST relationships. MOST is therefore widely used to parameterize boundary-layer processes. Such parameterizations are needed when modelling the effects of buoyancy on transport near the ground in weather models, LES formulations, etc.. This is the situation despite its many shortcomings.

The present questions are 'Why does it work as well as it does?' and 'Is there another, more defensible way to characterize the effects of stability on turbulent transport?'

2 Let's follow the energy

Turbulent flows are dissipative systems which, like all dissipative systems, run down and stop unless supported by a continuous supply of energy from outside the system. Let's focus on that energy.

For a mechanical system such as a turbulent flow, the relevant forms of energy are the various components of mechanical energy. Mechanical energy, E_{M}, is kinetic energy, E_{K}, plus forms of potential energy that can be converted to kinetic energy. For turbulent flows the relevant forms of energy are gravitational potential energy, E_{g}, which is associated with buoyancy, and pressure potential energy, often shortened to just pressure energy, E_{P}. Thus

$$E_{\mathrm{M}} = E_{\mathrm{K}} + E_{\mathrm{g}} + E_{\mathrm{P}} \tag{1}$$

In turbulence studies energy density is usually divided by fluid density, so it has units of velocity squared.

Mechanical energy is not conserved in dissipative systems. Rather, dissipation represents a loss since dissipated energy cannot be converted back into kinetic energy. We can write its material derivative as

$$\frac{\mathrm{D}}{\mathrm{D}t}(E_{\mathrm{K}} + E_{\mathrm{g}} + E_{\mathrm{P}}) = -\varepsilon \tag{2}$$

where ε is the dissipation rate. The minus sign on the right arises because ε is taken as positive when it represents a loss of mechanical energy.

Equation (2) is conceptually simple but not often written in this form. The turbulence community knows it better when transformed into an Eulerian frame of reference and rewritten with each variable divided into ensemble-averaged and deviation-from-average parts. It then becomes two equations, one for the mechanical energy of the mean flow, and the other for that associated with the fluctuating components. The latter is known as the Reynolds-averaged Navier-Stokes (RANS) energy equation, even while the Navier-Stokes equations are not necessary for its derivation. Equation (2) and the RANS energy equation rely simply on the definitions of mechanical energy and its components along with the principle of conservation of energy.

If we want to look at the effects of buoyancy on a flow then one way to do that is to look at the contribution that buoyancy makes to the supply of mechanical energy. We look at the ratio

$$\frac{\frac{\mathrm{D}}{\mathrm{D}t}E_{\mathrm{g}}}{\frac{\mathrm{D}}{\mathrm{D}t}(E_{\mathrm{K}} + E_{\mathrm{g}} + E_{\mathrm{P}})} \tag{3}$$

which is simplified greatly when we notice that the denominator is simply the dissipation rate. This ratio will, no doubt, be sensitive to the effects of buoyancy in any flow. If we apply (3) to the representative average for air parcels that cross a thin horizontal slice of the atmosphere above level ground then it can be written as

$$-\frac{g\,\overline{w'\theta'}}{\varepsilon\,\overline{\theta}} \tag{4}$$

where $\overline{w'\theta'}$ is the heat flux, and ε the dissipation rate at height z. If that slice lies within an atmospheric boundary layer then the heat flux flux decreases with height according to

$$\overline{w'\theta'} = (H_0 + H_i)\left(1 - \frac{z}{z_i}\right) \tag{5}$$

where H_0 and H_i are the heat flux at the ground and at inversion height, z_i, respectively.

Now let us move into MOST territory by considering relationships within the atmospheric surface layer at the base of a CBL. First assume z/z_i to be negligably small, as is consistent with the reasoning of Monin and Obukhov, and let H_i/H_0 be a constant, say 0.8, in the absence of a fuller treatment of the energy balance of the whole CBL. Next we write the dissipation rate as

$$\varepsilon = \frac{u_\varepsilon^3}{k\,z} \tag{6}$$

With these approximations and assumptions and neglecting the 0.8 factor, our energy flow ratio becomes

$$-\frac{k\,z\,g\,H_0}{u_\varepsilon^3\,\overline{\theta}} \tag{7}$$

This looks very like the MOST expression for $-z/L$, except that u_ε replaces u_*. We therefore define a modified Monin-Obukhov length, L_ε by

$$L_\varepsilon = -\frac{u_\varepsilon^3\,\overline{\theta}}{k\,g\,H_0} \tag{8}$$

An advantage of this length over the MOST length is that it continues to apply in the limit of windless convection. This is because the large eddies cause variable shear stresses, and so create dissipation near the ground even in that limit. That is, (7) takes account of this effect of the large eddies in the CBL while MOST does not. It provides an alternative to MOST and it works in free convection. Its rationality contrasts with the fix proposed by Beljaars (1994), who avoided a zero limit for L in free convection by adding a term proportional to Deardorff's outer convection velocity squared, w_*^2, when writing the drag relationship for u_*^2. Equation (6) does not describe dissipation above the surface friction layer, so (7) does not apply above that layer.

L_ε itself has some limitations. According to (5) it relies on z being small and the entrained heat flux being in fixed ratio to the surface heat flux. These assumptions create an uncertainty of at least 10% when used over a range of situations. Also, (7) applies only where (6) applies, which is to say in the dynamic and dynamic-convective sublayers of the surface layer, as designated by Kader and Yaglom (1990). These two layers combined have also been called the surface friction layer (SFL) because this is the layer where surface drag directly causes extra dissipation.

More generally, our definition of L_ε is based on the knowledge that energy flow is an important determinant of eddy properties in any turbulent flow, this together with information on how that energy flow depends on height in the surface friction layer. This does not guarantee that $-z/L_\varepsilon$ is sufficient to fully describe the turbulence at a given heigh since other factors, such as z_i or z_0, may also be involved. Further refinement would require further empirical information and/or modelling. By contrast, MOST simply asserts that no other parameters are involved beyond those included in its basis set, which set includes u_* but excludes u_ε.

3 Relationship between L_ε and z_s

Another route to deriving a scale length for the surface layer is by conceptual modelling of the turbulence processes themselves. This route was taken by (McNaughton, 2004; McNaughton et al, 2007) who proposed that the momentum-carrying eddies of the SFL are organized in upscale cascades of attached eddies, which structures compete for space as they grow, resulting in the self-similar turbulence properties observed in the log layer. These cascades grow until disrupted by outer eddies supported by a stronger flow of mechanical energy in the outer layer, which flow equals the outer dissipation rate, ε_o.

The outer dissipation rate is observed to be constant with height through the CBL above the SFL. The height of the SFL is then given by

$$z_s = \frac{u_\varepsilon^3}{k\varepsilon_o} \tag{9}$$

so the height of the SFL, z_s becomes an important length scale when describing turbulence processes within the SFL.

The ratio of z_s to the traditional Obukhov length is given by

$$-\frac{z_s}{L} = \frac{u_*^3}{u_\varepsilon^3} \tag{10}$$

We know that $-z/L$ can characterize the height of the SFL fairly well, with $1 < -z/L < 2$ at the transition in several experiments, so $0.5 < -z_s/L < 1$. Clearly the ratio $-z_s/L$ is not constant but depends on the effects of the large eddies via the ratio on the right of (10), being larger in nearer-neutral conditions and smaller in more convective conditions. This dependency explains the observed scatter when z_s is plotted against L, as reported by Chowdhuri and McNaughton (2019).

Perhaps L_ε can do better than L? The ratio of z_s to the modified Obukhov length, L_ε, is given by

$$-\frac{z_s}{L_\varepsilon} = \frac{g\,H_0}{\varepsilon_o\,\overline{\theta}} \tag{11}$$

so L_ε also correlates with z_s. If z_s and L_ε are to be perfectly correlated then ε_o must be proportional to $g\,H_0/\overline{\theta}$. To investigate such a relationship we first look at the Deardorff convective velocity scale, defined by

$$w_*^3 = \frac{z_i\,g\,H_0}{\overline{\theta}} \tag{12}$$

Substitution into (11) then gives

$$-\frac{z_s}{L_\varepsilon} = \frac{w_*^3}{z_i\,\varepsilon_o} \tag{13}$$

We note that $(z_i\varepsilon_o)^{1/3}$ is closely related to the outer velocity scale $(\lambda\varepsilon_o)^{1/3}$ defined by Chowdhuri and McNaughton (2019) so, to the extent that w_* is that same velocity scale to within a constant $\mathcal{O}(1)$ the right hand side of (13) should be a constant $\mathcal{O}(1)$. There have been no empirical investigations to confirm this.

Another approach is to look at the empirical investigations of Kader and Yaglom (1990). They find that

$$\varepsilon_o = 1.1\,g\,H_0/\overline{\theta} \tag{14}$$

in the convective sublayer. This relationship was confirmed, with reduced scatter, by Chowdhuri and McNaughton (2019). Substituting this into (11) gives

$$-\frac{z_s}{L_\varepsilon} = 1.1 \tag{15}$$

Some caution should be expressed here. If we consider (14) in the context of the mechanical energy budget of the whole CBL then it says that more mechanical energy is being dissipated in the CBL than is delivered as gravitational potential energy. That is unreasonable in free-convection conditions since there can be no entrainment of kinetic energy in such conditions. Allowing also for some work to be done against buoyancy during entrainment, the constant in (15) should be a little less than one in this limit. This assessment supposes that H_0 was measured accurately in the Tsimlyansk and SLTEST studies. However, H_0 was measured by eddy covariance in both studies, which method typically underestimates fluxes by about 15% (Massman and Lee, 2002). In less convective conditions, as the contribution of entrained kinetic energy increases, the height of the SFL increases and extra dissipation in that layer cannot be neglected. We note that both Kader and Yaglom (1990) and Chowdhuri and McNaughton (2019) analysed data from rather highly convective conditions, so (15), while encouraging as it stands, it requires further investigation.

Clearly there is scope for a new analysis of the energy budgets of CBL flows over a wider range of stabilities. The Tsimlyansk and SLTEST experiments were both limited by their abilities to measure ε_o directly, which is to say by the heights of their towers. The tower at Tsimlyansk was 80m over grasslands (with $z_0 \sim .02$m) and at SLTEST the top instrument was at 25.7 m over a smooth playa surface (with $z_0 \sim .0005$m). Also, neither experiment measured z_i directly, so neither could observe the relationship between z_i and λ.

4 Conclusions

MOST works as well as it does because it gets some things right. It follows Richardson (1920) in identifying the surface heat flux as an important influence on turbulence properties near the ground, and it identifies the surface shear layer as the environment in which those properties are expressed. What it gets wrong is that it also follows Richardson by neglecting the effects of the upper boundary to the flow, and the way that energy flows down through the outer flow to influence turbulence properties near the ground.

There is another way to characterize the effects of stability on turbulent transport, and that way puts the energy flow through the boundary-layer system at its conceptual centre. This can be done either by modelling the ratio of the flow of gravitational potential energy to total mechanical energy, either directly and empirically or in the context of a conceptual model of turbulence processes near the ground. It seems that both give similar results for the surface friction layer beneath a convective atmospheric boundary layer. The conceptual model of McNaughton (2004); McNaughton et al (2007) also interprets its length scale, z_s, as the physical height of the surface friction layer.

We have addressed the surface friction layers beneath CBLs. The length scale L_ε may also be useful in near-neutral ABLs in which the surface shear layer extends upwards to occupy the whole NNBL, so z_s has no meaning in NNBLs. Also, u_ε approaches u_* as neutrality is approached, so unmodified MOST should work somewhat better in NNBLs than in CBLs. Perhaps this is the reason why the Kansas spectra, which were binned according to z/L values before averaging, were fairly orderly in near-neutral conditions out to $-z/L = 0.3$, but rather disorderly at greater instabilities.

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